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Reliability Sensitivity of Multi-Degree-of-Freedom Uncertain Nonlinear Systems with Independent Failure Modes

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Abstract

The reliability sensitivity analysis of uncertain multi-degree-of-freedom nonlinear vibration systems with independent failure modes subjected to random excitation is examined. An earlier version of the statistical fourth-moment method is extended to deal with vector-valued and matrix-valued functions and is developed to determine the first four moments of the system response and state function. Random variables and system derivatives are conveniently arranged into 2D matrices by means of Kronecker algebra. The distribution function of the system state function is approximately determined by the standard normal distribution functions using Edgeworth series technique, and its reliability and reliability sensitivity are presented and discussed.

Keywords: Reliability; Reliability sensitivity; Nonlinear system

1. Introduction

Uncertainties in material properties and structural geometry are due to measurement inaccuracies or structure complexities. Randomness may be mathematically modeled either as a random variable or a stochastic process. The study of reliability analysis of uncertain systems is important for design purposes. The reliability analysis can help the designer to establish acceptable tolerances on structural systems. Parameter uncertainty is inherent in most engineering problems. Its effects on structural response and reliability should be assessed. The reliability problem has been addressed in number of publications (Ang and Tang, 1975 ; Haldar and Mahadevan, 2000 ; Zhang *et al.*, 1998 ; 2002 ; 2005a). A set useful sensitivity analy-sis in vibration reliability has practical applications within reliability-based design, in optimization of structural design, construction, maintenance and inspection under reliability constraints, in parameter studies of the reliability, and in reliability updating. Structural reliability sensitivity calculation methods are well developed (Madsen *et al.*, 1986 ; Zhang *et al.*, 2003 ; 2005b). These publications have presented the efficient and accurate computational reliability sensi-tivity methods.

This paper presents an approximate solution technique of reliability sensitivity for general multi-degree-of-freedom nonlinear random vibration systems with random parameters. The associated reliability and reliability sensitivity function of some random nonlinear vibration systems with uncorrelated failure modes are solv.ed. This sophisticated formulation is easily amenable to computational procedures.

2. System response

It is known that the nonlinear equations of motion of in structural vibrations are

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$$\boldsymbol{M}\ddot{\boldsymbol{x}}+\boldsymbol{f}(\boldsymbol{x},\dot{\boldsymbol{x}})=\boldsymbol{F}(t) \tag{1}$$

When the structural parameters are uncertain, the structural system governed by the following nonlinear equations is considered

$$\boldsymbol{M}(\boldsymbol{B})\ddot{\boldsymbol{x}}(\boldsymbol{B},t)+\boldsymbol{f}(\boldsymbol{B},\boldsymbol{x},\dot{\boldsymbol{x}})=\boldsymbol{F}(\boldsymbol{B},t) \tag{2}$$

where the probabilistic effects are described through the random parameter matrix $\boldsymbol{B}=(b_{ij})_{s\times t}$ of order $s\times t$, which contains all random variables.

Suppose $\boldsymbol{B}(s\times t)$ and $\boldsymbol{A}(p\times 1)$ are matrices of dimension $s\times t$ and dimension $p\times 1$ of random variables related by the transformation

$$\boldsymbol{A}=\boldsymbol{A}(\boldsymbol{B}) \tag{3}$$

Then the second order Taylor expansion of $\boldsymbol{A}$ (Vetter, 1973) about a nominal value $\overline{\boldsymbol{B}}$ of $\boldsymbol{B}$ is given

$$\boldsymbol{A}(\boldsymbol{B})=\boldsymbol{A}(\overline{\boldsymbol{B}})+\left.\frac{\partial \boldsymbol{A}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right|_{\boldsymbol{B}=\overline{\boldsymbol{B}}}\mathrm{d}[\mathrm{cs}(\boldsymbol{B})] +\frac{1}{2}\left.\frac{\partial^2 \boldsymbol{A}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}2}}\right|_{\boldsymbol{B}=\overline{\boldsymbol{B}}}\{\mathrm{d}[\mathrm{cs}(\boldsymbol{B})]\}^{[2]} \tag{4}$$

The matrices of the both sides of Eq. (2) are expanded about $\overline{\boldsymbol{B}}$ via Taylor series

$$\begin{aligned}\boldsymbol{M}\ddot{\boldsymbol{x}}=&\overline{\boldsymbol{M}}\ddot{\overline{\boldsymbol{x}}}+\left[\frac{\partial\overline{\boldsymbol{M}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}(\boldsymbol{I}_{st}\otimes\ddot{\overline{\boldsymbol{x}}})+\overline{\boldsymbol{M}}\frac{\partial\ddot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]\mathrm{d}(\mathrm{cs}\boldsymbol{B})\\&+\frac{1}{2}\left[\frac{\partial^2\overline{\boldsymbol{M}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}2}}(\boldsymbol{I}_{s^2t^2}\otimes\ddot{\overline{\boldsymbol{x}}})+\frac{\partial\overline{\boldsymbol{M}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\frac{\partial\ddot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\otimes\boldsymbol{I}_{st}\right)\right.\\&\left.+\frac{\partial\overline{\boldsymbol{M}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\boldsymbol{I}_{st}\otimes\frac{\partial\ddot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right)+\overline{\boldsymbol{M}}\frac{\partial^2\ddot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}2}}\right][\mathrm{d}(\mathrm{cs}\boldsymbol{B})]^{[2]}\end{aligned} \tag{5}$$

$$\boldsymbol{F}=\overline{\boldsymbol{F}}+\frac{\partial\overline{\boldsymbol{F}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\mathrm{d}(\mathrm{cs}\boldsymbol{B})+\frac{1}{2}\frac{\partial^2\overline{\boldsymbol{F}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}2}}[\mathrm{d}(\mathrm{cs}\boldsymbol{B})]^{[2]} \tag{6}$$

and

$$\begin{aligned}\boldsymbol{f}=&\overline{\boldsymbol{f}}+\left[\frac{\partial\overline{\boldsymbol{f}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}+\overline{\boldsymbol{C}}\frac{\partial\dot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}+\overline{\boldsymbol{K}}\frac{\partial\overline{\boldsymbol{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]\mathrm{d}(\mathrm{cs}\boldsymbol{B})\\&+\frac{1}{2}\left[\frac{\partial^2\overline{\boldsymbol{f}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}2}}+\frac{\partial\overline{\boldsymbol{C}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\frac{\partial\dot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\otimes\boldsymbol{I}_{st}\right)\right.\\&+\frac{\partial\overline{\boldsymbol{K}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\frac{\partial\overline{\boldsymbol{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\otimes\boldsymbol{I}_{st}\right)+\frac{\partial\overline{\boldsymbol{C}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\boldsymbol{I}_{st}\otimes\frac{\partial\dot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right)\\&+\frac{\partial\overline{\boldsymbol{K}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\boldsymbol{I}_{st}\otimes\frac{\partial\overline{\boldsymbol{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right)+\overline{\boldsymbol{C}}\frac{\partial^2\dot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}2}}\\&\left.+\overline{\boldsymbol{K}}\frac{\partial^2\overline{\boldsymbol{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}2}}\right][\mathrm{d}(\mathrm{cs}\boldsymbol{B})]^{[2]}\end{aligned} \tag{7}$$

Substituting Eqs. (5)-(7) into Eq. (2) and equating similar order terms, one has the zeroth-order, first-order, second-order equations corresponding to Eq. (2):

Zeroth order

$$\overline{\boldsymbol{M}}\ddot{\overline{\boldsymbol{x}}}+\overline{\boldsymbol{f}}=\overline{\boldsymbol{F}} \tag{8}$$

First order (ε terms)

$$\overline{\boldsymbol{M}}\ddot{\boldsymbol{x}}_1+\overline{\boldsymbol{C}}\dot{\boldsymbol{x}}_1+\overline{\boldsymbol{K}}\boldsymbol{x}_1=\boldsymbol{F}_1 \tag{9}$$

and

$$\boldsymbol{F}_1=\left[\frac{\partial\overline{\boldsymbol{F}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}-\frac{\partial\overline{\boldsymbol{f}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}-\frac{\partial\overline{\boldsymbol{M}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}(\boldsymbol{I}_{st}\otimes\ddot{\overline{\boldsymbol{x}}})\right][\mathrm{cs}(\boldsymbol{B}-\overline{\boldsymbol{B}})] \tag{10}$$

Second order (ε^2 terms)

$$\overline{\boldsymbol{M}}\ddot{\boldsymbol{x}}_2+\overline{\boldsymbol{C}}\dot{\boldsymbol{x}}_2+\overline{\boldsymbol{K}}\boldsymbol{x}_2=\overline{\boldsymbol{F}}_2 \tag{11}$$

and

$$\begin{aligned}\overline{\boldsymbol{F}}_2=&\frac{1}{2}\left[\frac{\partial^2\overline{\boldsymbol{F}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}2}}-\frac{\partial^2\overline{\boldsymbol{f}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}2}}-\frac{\partial^2\overline{\boldsymbol{M}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}2}}(\boldsymbol{I}_{s^2t^2}\otimes\ddot{\overline{\boldsymbol{x}}})\right.\\&-\frac{\partial\overline{\boldsymbol{M}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\frac{\partial\ddot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\otimes\boldsymbol{I}_{st}\right)-\frac{\partial\overline{\boldsymbol{M}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\boldsymbol{I}_{st}\otimes\frac{\partial\ddot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right)\\&-\frac{\partial\overline{\boldsymbol{C}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\frac{\partial\dot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\otimes\boldsymbol{I}_{st}\right)-\frac{\partial\overline{\boldsymbol{C}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\boldsymbol{I}_{st}\otimes\frac{\partial\dot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right)\\&-\frac{\partial\overline{\boldsymbol{K}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\frac{\partial\overline{\boldsymbol{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\otimes\boldsymbol{I}_{st}\right)\\&\left.-\frac{\partial\overline{\boldsymbol{K}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\left(\boldsymbol{I}_{st}\otimes\frac{\partial\overline{\boldsymbol{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right)\right][\mathrm{Var}(\mathrm{cs}\boldsymbol{B})]\end{aligned} \tag{12}$$

Once $\ddot{\overline{\boldsymbol{x}}},\dot{\overline{\boldsymbol{x}}},\overline{\boldsymbol{x}}$, and $\ddot{\overline{\boldsymbol{x}}}_2,\dot{\overline{\boldsymbol{x}}}_2,\overline{\boldsymbol{x}}_2$ are obtained, and $\ddot{\boldsymbol{x}}_1,\dot{\boldsymbol{x}}_1,\boldsymbol{x}_1$ can be determined. The mean value, variance, third-order moment and fourth-order moment matrices of the responses can then be computed.

According to the fourth-moment technique, the mean value, variance, third-order moment and fourth-order moment of the responses are represented as

$$\mathrm{E}(\ddot{\boldsymbol{x}})=\ddot{\overline{\boldsymbol{x}}}+\ddot{\overline{\boldsymbol{x}}}_2 \tag{13}$$

$$\mathrm{E}(\dot{\boldsymbol{x}})=\dot{\overline{\boldsymbol{x}}}+\dot{\overline{\boldsymbol{x}}}_2 \tag{14}$$

$$\mathrm{E}(\boldsymbol{x})=\overline{\boldsymbol{x}}+\overline{\boldsymbol{x}}_2 \tag{15}$$

$$\mathrm{Var}(\ddot{\boldsymbol{x}})=\mathrm{E}(\ddot{\boldsymbol{x}}_1\otimes\ddot{\boldsymbol{x}}_1)=\left[\frac{\partial\ddot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]^{[2]}[\mathrm{Var}(\mathrm{cs}\boldsymbol{B})] \tag{16}$$

$$\mathrm{Var}(\dot{\boldsymbol{x}})=\mathrm{E}(\dot{\boldsymbol{x}}_1\otimes\dot{\boldsymbol{x}}_1)=\left[\frac{\partial\dot{\overline{\boldsymbol{x}}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]^{[2]}[\mathrm{Var}(\mathrm{cs}\boldsymbol{B})] \tag{17}$$

$$\mathrm{Var}(\boldsymbol{x})=\mathrm{E}(\boldsymbol{x}_1\otimes\boldsymbol{x}_1)=\left[\frac{\partial\overline{\boldsymbol{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]^{[2]}[\mathrm{Var}(\mathrm{cs}\boldsymbol{B})] \tag{18}$$

$$\mathrm{M3}(\ddot{x}) = \mathrm{E}(\ddot{x}_1 \otimes \ddot{x}_1 \otimes \ddot{x}_1) = \left[\frac{\partial \ddot{\bar{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]^{[3]} [\mathrm{M3}(\mathrm{cs}\boldsymbol{B})] \tag{19}$$

$$\mathrm{M3}(\dot{x}) = \mathrm{E}(\dot{x}_1 \otimes \dot{x}_1 \otimes \dot{x}_1) = \left[\frac{\partial \dot{\bar{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]^{[3]} [\mathrm{M3}(\mathrm{cs}\boldsymbol{B})] \tag{20}$$

$$\mathrm{M3}(x) = \mathrm{E}(x_1 \otimes x_1 \otimes x_1) = \left[\frac{\partial \bar{x}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]^{[3]} [\mathrm{M3}(\mathrm{cs}\boldsymbol{B})] \tag{21}$$

$$\mathrm{M4}(\ddot{x}) = \mathrm{E}(\ddot{x}_1 \otimes \ddot{x}_1 \otimes \ddot{x}_1 \otimes \ddot{x}_1) = \left[\frac{\partial \ddot{\bar{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]^{[4]} [\mathrm{M4}(\mathrm{cs}\boldsymbol{B})] \tag{22}$$

$$\mathrm{M4}(\dot{x}) = \mathrm{E}(\dot{x}_1 \otimes \dot{x}_1 \otimes \dot{x}_1 \otimes \dot{x}_1) = \left[\frac{\partial \dot{\bar{x}}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]^{[4]} [\mathrm{M4}(\mathrm{cs}\boldsymbol{B})] \tag{23}$$

$$\mathrm{M4}(x) = \mathrm{E}(x_1 \otimes x_1 \otimes x_1 \otimes x_1) = \left[\frac{\partial \bar{x}}{\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}}\right]^{[4]} [\mathrm{M4}(\mathrm{cs}\boldsymbol{B})] \tag{24}$$

In order to get the derivations $\partial\ddot{\bar{x}}/\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}$, $\partial\dot{\bar{x}}/\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}$, $\partial\bar{x}/\partial(\mathrm{cs}\boldsymbol{B})^{\mathrm{T}}$, one obtains the sensitivity equations from Eq. (9). Once, the sensitivities can be deter-mined. The matrices, $\mathrm{Var}(\ddot{x}), \mathrm{Var}(\dot{x}), \mathrm{Var}(x)$, $\mathrm{M3}(\ddot{x})$, $\mathrm{M3}(\dot{x}), \mathrm{M3}(x)$, $\mathrm{M4}(\ddot{x}), \mathrm{M4}(\dot{x}), \mathrm{M4}(x)$ of $\ddot{x}, \dot{x}, x$ are obtained.

3. System reliability

The first passage problem for the multi-degree-of-freedom nonlinear vibration system with displacement tolerance is defined by

$$g_i(G_i, x_i) = |G_i| - |x_i| \tag{25}$$

where the response $\boldsymbol{x}$ and the threshold $\boldsymbol{G}$ are mutual independent random variables.

The reliability index is defined by (Ang, and Tang, 1975)

$$\beta_i = \frac{\mathrm{E}(g_i)}{\sqrt{\mathrm{Var}(g_i)}} = \frac{\mu_{g_i}}{\sigma_{g_i}} \tag{26}$$

The arbitrary distribution function of the standard random variables that is approximately expressed by the standard normal distribution function using Edgeworth series is addressed in (Cramer, 1964)

$$F(y_i) = \Phi(y_i) - \varphi(y_i)\left[\frac{1}{6}\frac{\theta_{g_i}}{\sigma_{g_i}^3} H_2(y_i) + \frac{1}{24}\left(\frac{\eta_{g_i}}{\sigma_{g_i}^4} - 3\right) H_3(y_i) + \frac{1}{72}\left(\frac{\theta_{g_i}}{\sigma_{g_i}^3}\right)^2 H_5(y_i)\right] \tag{27}$$

Thus, the reliability of the system is given by

$$R_i = P[g_i(G_i, x_i) > 0] = 1 - F(-\beta_i) \tag{28}$$

When Eq. (28) is used to determined the reliability of the system. It is possible that R_i>1. To eliminate such situation, the following definition (Zhang *et al.*, 1998) will be used

$$R_i^* = R(\beta_i) - \left\{\frac{R(\beta_i) - \Phi(\beta_i)}{[1 + (R(\beta_i) - \Phi(\beta_i))\beta_i]^{\beta_i}}\right\} \tag{29}$$

4. Reliability sensitivity

The reliability sensitivity with respect to the mean value and the standard variance of the system response is derived as follow respectively.

$$\begin{aligned}\frac{\mathrm{D}R_i}{\mathrm{D}\mu_{x_i}} = -\frac{\varphi(\beta_i)}{\sigma_{g_i}} \Bigg\{ & 1 + \beta_i \left[\frac{1}{6}\frac{\theta_{g_i}}{\sigma_{g_i}^3} H_2(\beta_i) + \frac{1}{24}\left(\frac{\eta_{g_i}}{\sigma_{g_i}^4} - 3\right) H_3(\beta_i) + \frac{1}{72}\left(\frac{\theta_{g_i}}{\sigma_{g_i}^3}\right)^2 H_5(\beta_i)\right] \\ & - \left[\frac{1}{3}\frac{\theta_{g_i}}{\sigma_{g_i}^3} H_1(\beta_i) + \frac{1}{8}\left(\frac{\eta_{g_i}}{\sigma_{g_i}^4} - 3\right) H_2(\beta_i) + \frac{5}{72}\left(\frac{\theta_{g_i}}{\sigma_{g_i}^3}\right)^2 H_4(\beta_i)\right]\Bigg\}\end{aligned} \tag{30}$$

$$\begin{aligned}\frac{\mathrm{D}R_i}{\mathrm{D}\sigma_{x_i}} = -\frac{2\mu_{g_i}\sigma_{x_i}}{\sigma_{g_i}^2} \varphi(\beta_i) \Bigg\{ & 1 + \beta_i \left[\frac{1}{6}\frac{\theta_{g_i}}{\sigma_{g_i}^3} H_2(\beta_i) + \frac{1}{24}\left(\frac{\eta_{g_i}}{\sigma_{g_i}^4} - 3\right) H_3(\beta_i) + \frac{1}{72}\left(\frac{\theta_{g_i}}{\sigma_{g_i}^3}\right)^2 H_5(\beta_i)\right] \\ & - \left[\frac{1}{3}\frac{\theta_{g_i}}{\sigma_{g_i}^3} H_1(\beta_i) + \frac{1}{8}\left(\frac{\eta_{g_i}}{\sigma_{g_i}^4} - 3\right) H_2(\beta_i) + \frac{5}{72}\left(\frac{\theta_{g_i}}{\sigma_{g_i}^3}\right)^2 H_4(\beta_i)\right]\Bigg\} \\ & + 2\sigma_{x_i}\varphi(\beta_i)\left[\frac{1}{2}\frac{\theta_{g_i}}{\sigma_{g_i}^4} H_2(\beta_i) + \frac{1}{6}\frac{\eta_{g_i}}{\sigma_{g_i}^5} H_3(\beta) + \frac{1}{12}\frac{\theta_{g_i}^2}{\sigma_{g_i}^7} H_5(\beta)\right] \\ & - \frac{\sigma_{G_i}^2\sigma_{x_i}}{2\sigma_{g_i}^4} \varphi(\beta_i) H_3(\beta_i)\end{aligned} \tag{31}$$

5. Numerical example

Consider a two degree-of-freedom impacting system subjected to a horizontal acceleration

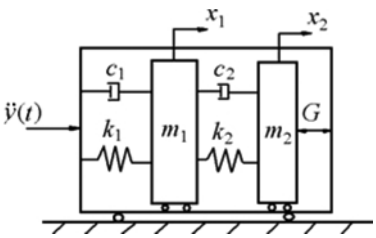

Fig. 1. System model.

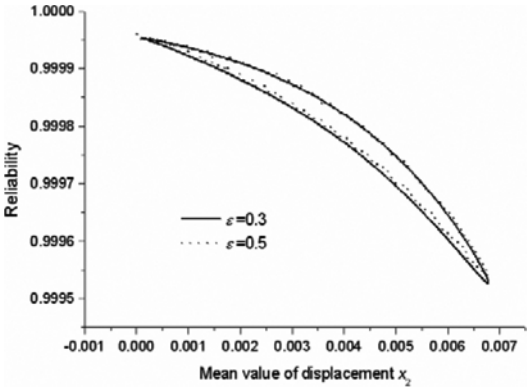

Fig. 2. The reliability with the mean value of the system response x_2.

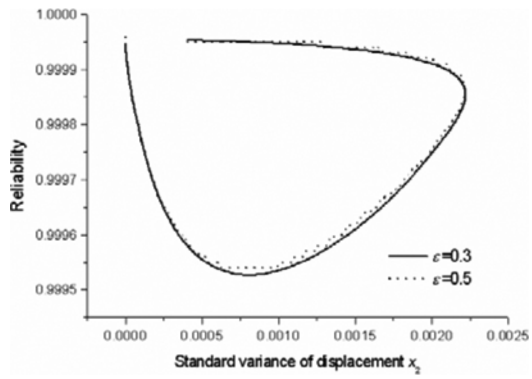

Fig. 3. The reliability with the standard variance of system response x_2.

$$
\begin{aligned}
&\ddot{y}(t) = 0.25t \ (\mathrm{cm/s^2}) && 0 \le t < 1 \\
&\ddot{y}(t) = 0.25(2-t) \ (\mathrm{cm/s^2}) && 1 \le t < 2 \\
&\ddot{y}(t) = 0 && \text{others}
\end{aligned}
\tag{32}
$$

as shown in Fig. 1 There is a gap G between the mass m_2 and the case wall. The stochastic differential equation is represented by

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} + \boldsymbol{K}_\varepsilon(\boldsymbol{x} \otimes \boldsymbol{x} \otimes \boldsymbol{x}) = \boldsymbol{F}(t) \tag{33}$$

with initial conditions $\boldsymbol{x}(0) = 0, \ \dot{\boldsymbol{x}}(0) = 0$ in which the deterministic masses m_1=2m_2=2 kg and damping

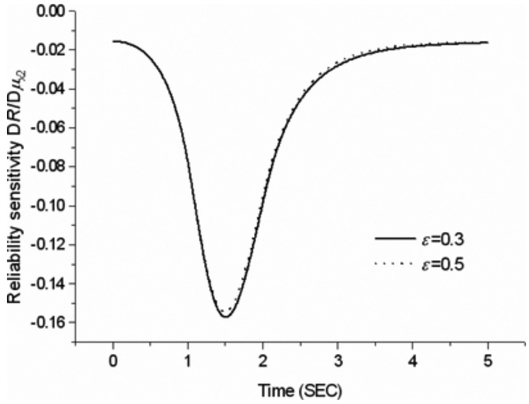

Fig. 4. The reliability sensitivity $\mathrm{D}R/\mathrm{D}\mu_{x_2}$ with respect to the mean value of the system response x_2.

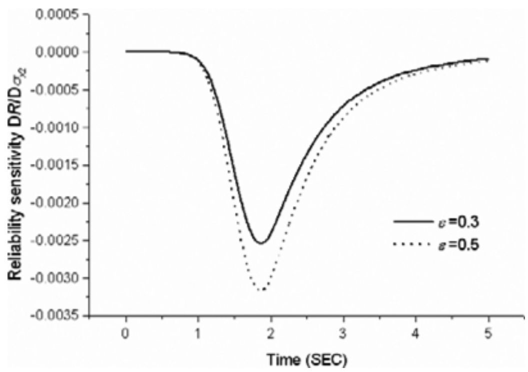

Fig. 5. The reliability sensitivity $\mathrm{D}R/\mathrm{D}\sigma_{x_2}$ with respect to the standard variance of the system response x_2.

coefficients c_1=c_2=75 Ns/cm The random parameters k_1, k_2 and G are mutual independent. The random spring constants k_1 and k_2 are normally distributed with a coefficient of variation equal to 0.05. The mean spring constants are 75 N/cm. The first four order moments of the gap G are found as E(G)=0.152 cm, Var(G)=3.92×10^{-4} cm^2, M3(G)=1.17×10^{-2} cm^3, M4(G)=6.9×10^{-3} cm^4 respectively. The random parameter matrix is $\boldsymbol{B}=[k_1\ k_2\ G]^{\mathrm{T}}$.

The reliabilities R with the mean value and the standard variance of the system response x_2 are depicted in Figure 2 and Figure 3. The reliability sensitivities $\mathrm{D}R/\mathrm{D}\mu_{x_2}$ and $\mathrm{D}R/\mathrm{D}\sigma_{x_2}$ are depicted in Figs. 4 and 5.

This example does not give the joint probability density or distribution functions, but only the first four moments. The method is efficient in computational probabilistic mechanics. A major advantage of these techniques is that the joint probability density or distribution functions need not be known.

6. Conclusions

The numerical formulation of the reliability sensitivity of the multi-degree-of-freedom nonlinear vibration system with the random parameters assuming uncorrelated failure modes subjected to the random excitation using the fourth-moment techniques and the Edgeworth series has been given. The method can modify demands for the distribution function of the random parameters and the excitations. The method is useful in reliability design and reliability optimization design of structural systems. It is new in the proposed methodology compared to previous work in reliability sensitivity for multi-degree-of-freedom nonlinear random vibration systems with random parameters.

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